LECTURE NO 6

Topics

- Differential length area and volume,
- line surface and volume integrals,
- del operator,
- gradient of a scalar

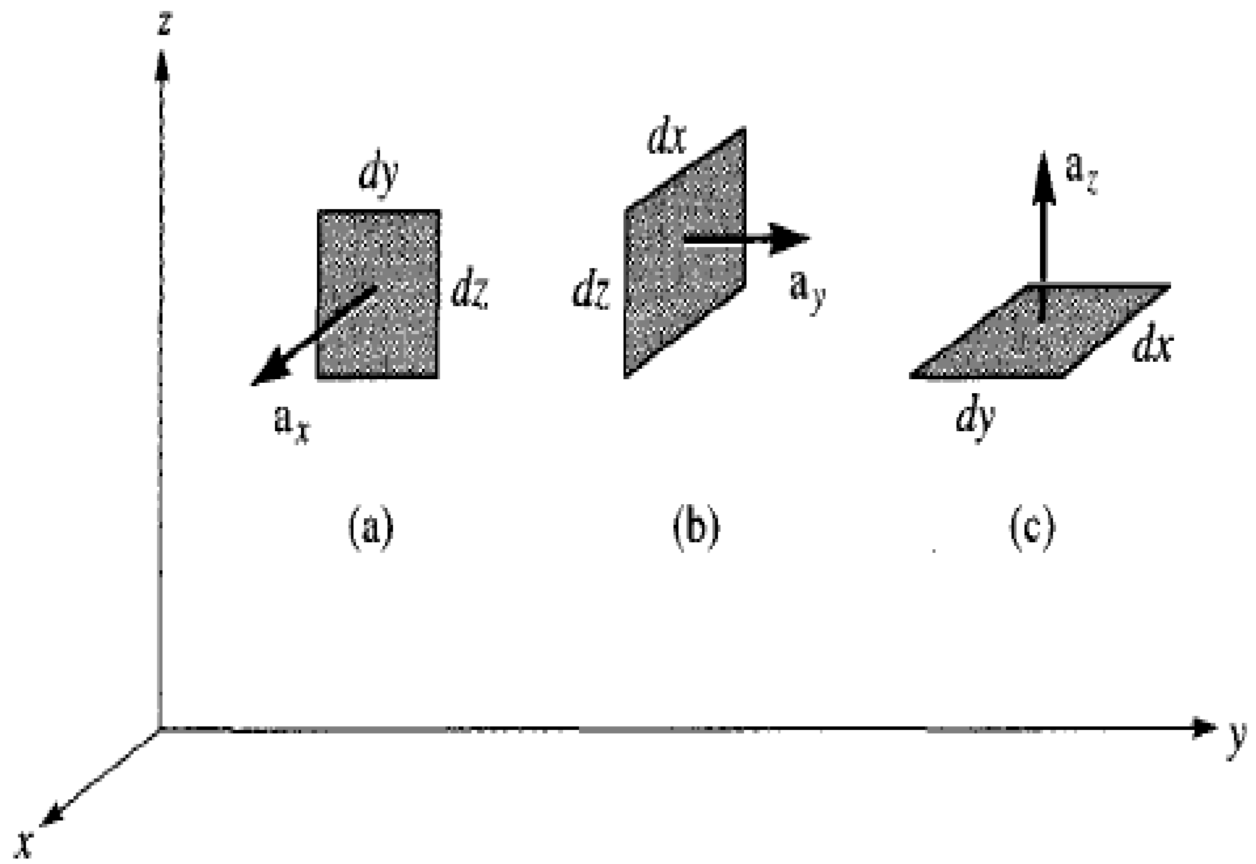
DIFFERENTIAL LENGTH, AREA, AND VOLUME

A. Cartesian Coordinates

$$d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$$

$$d\mathbf{S} = \begin{array}{l} dy\,dz\,\mathbf{a}_x \\ dx\,dz\,\mathbf{a}_y \\ dz\,dy\,\mathbf{a}_z \end{array}$$

$$dv = dx\,dy\,dz$$



(a) $d\mathbf{S} = dy\, dz\, \mathbf{a}_x$, **(b)** $d\mathbf{S} = dx\, dz\, \mathbf{a}_y$, **(c)** $d\mathbf{S} = dx\, dy\, \mathbf{a}_z$

Cylindrical Coordinates

(1) Differential displacement is given by

$$d\mathbf{l} = d\rho\, \mathbf{a}_\rho + \rho\, d\phi\, \mathbf{a}_\phi + dz\, \mathbf{a}_z$$

(2) Differential normal area is given by

$$d\mathbf{S} = \begin{matrix} \rho\, d\phi\, dz\, \mathbf{a}_\rho \\ d\rho\, dz\, \mathbf{a}_\phi \\ \rho\, d\phi\, d\rho\, \mathbf{a}_z \end{matrix}$$

(3) Differential volume is given by

$$dv = \rho\, d\rho\, d\phi\, dz$$

Spherical Coordinates

(1) The differential displacement is

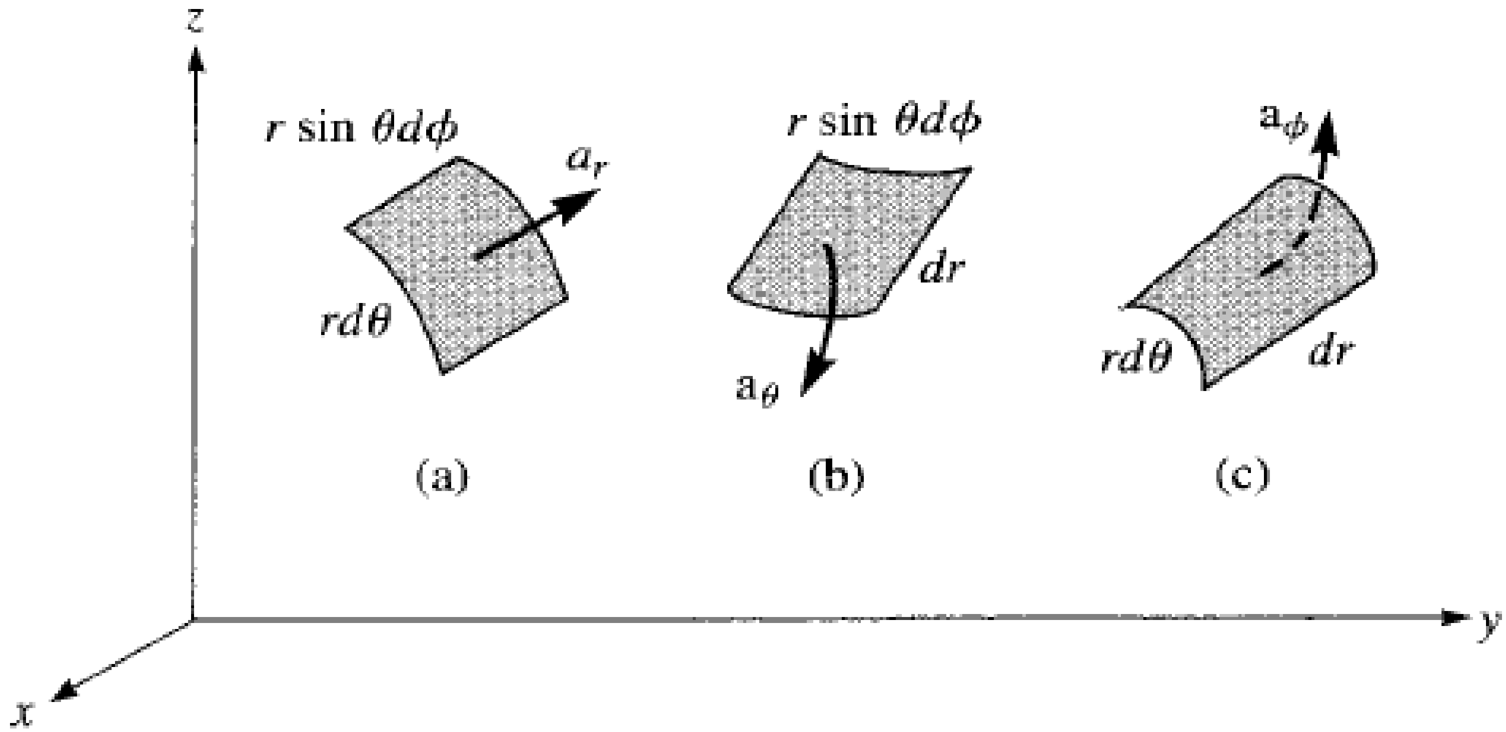
$$d\mathbf{l} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi$$

(2) The differential normal area is

$$\begin{aligned} d\mathbf{S} = {} & r^2 \sin\theta\,d\theta\,d\phi\,\mathbf{a}_r \\ & r\sin\theta\,dr\,d\phi\,\mathbf{a}_\theta \\ & r\,dr\,d\theta\,\mathbf{a}_\phi \end{aligned}$$

(3) The differential volume is

$$dv = r^2 \sin\theta\,dr\,d\theta\,d\phi$$



(a) $d\mathbf{S} = r^2 \sin\theta \, d\theta \, d\phi \, \mathbf{a}_r$, **(b)** $d\mathbf{S} = r \sin\theta \, dr \, d\phi \, \mathbf{a}_\phi$,
(c) $d\mathbf{S} = r \, dr \, d\theta \, \mathbf{a}_\phi$